Week 4
MATH 4A
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4-2.3 Let $e_1 = (1,0)$, $e_2 = (0,1)$, $x_1 = (4,5)$, and $x_2 = (-7,5)$. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation that sends $e_1 \mapsto x_1$ and $e_2 \mapsto x_2$. What is $T(-8,3)$?

$T(e_1) = x_1 = (4,5)$
$T(e_2) = x_2 = (-7,5)$

$$
\begin{aligned}
T(-8,3) &= T(-8e_1 + 3e_2) \\
&= T(-8e_1) + T(3e_2) \\
&= -8\,T(e_1) + 3T(e_2) \\
&= -8(4,5) + 3(-7,5) \\
&= (-32,-40) + (-21,15) \\
&= (-53,-25)
\end{aligned}
$$

4-2.5 Let $v_1 = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$. Suppose $T(v_1) = \begin{bmatrix} -12 \\ 8 \end{bmatrix}$ and $T(v_2) = \begin{bmatrix} 19 \\ -9 \end{bmatrix}$. For an arbitrary vector $v = \begin{bmatrix} x \\ y \end{bmatrix}$, find $T(v)$.

If we can express $V = C_1 V_1 + C_2 V_2$, then

$$T(v) = C_1 T(V_1) + C_2 T(V_2)$$

So, we first find C_1, C_2 such that $V = C_1 V_1 + C_2 V_2$.

$$\Updownarrow$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = C_1 \begin{bmatrix} -1 \\ -2 \end{bmatrix} + C_2 \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} -1 & 1 & x \\ -2 & 3 & y \end{array}\right]$$

$$\Big\downarrow \text{ RREF} \ldots$$

$$\left[\begin{array}{cc|c} 1 & 0 & -3x+y \\ 0 & 1 & -2x+y \end{array}\right]$$

So, $C_1 = -3x+y$, $C_2 = -2x+y$.

So, $T(v) = C_1 T(V_1) + C_2 T(V_2)$

$$= (-3x+y) \begin{bmatrix} -12 \\ 8 \end{bmatrix} + (-2x+y) \begin{bmatrix} 19 \\ -9 \end{bmatrix}$$

$$= \begin{bmatrix} -2x+7y \\ -6x-y \end{bmatrix} //$$

4-2.7 Given $T : \mathbb{R}^2 \to \mathbb{R}^2$ such that $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$ and $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$. Find the matrix A of (ie. that represents) T.

$$e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} T(e_1) & T(e_2) \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 \\ -1 & -3 \end{bmatrix}$$

1. Let $T : \mathbb{R}^r \to \mathbb{R}^s$. Determine whether or not T is onto in each of the following situations:

   (a) $r = s$

   (b) $r < s$

   (c) $r > s$

a) Not enough info.

Consider $T: \mathbb{R}^2 \to \mathbb{R}^2$ where $\begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} x \\ y \end{bmatrix}$ (ie. identity map)

Clearly onto...

On the other hand, consider $T: \mathbb{R}^2 \to \mathbb{R}^2$ where $\begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ (ie. zero map)

Clearly <u>NOT</u> onto, since only $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ is mapped to.

b) Not enough info.

Consider $T: \mathbb{R}^3 \to \mathbb{R}^2$ where $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \mapsto \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \mapsto \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \mapsto \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

(ie. $\begin{bmatrix} x \\ y \\ z \end{bmatrix} \mapsto \begin{bmatrix} x \\ y \end{bmatrix}$)

Easy to see onto.

But, as before, $T: \mathbb{R}^3 \to \mathbb{R}^2$ where $\begin{bmatrix} x \\ y \\ z \end{bmatrix} \mapsto \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

<u>NOT</u> onto.

c) Not (ie. <u>Never</u>) onto!

Given $T: \mathbb{R}^r \to \mathbb{R}^s$, the image of T is at <u>most</u> r. If $r < s$, then dimension of image always smaller than dim. of target space (codomain).

dimension